

# $\phi^4$ inflation is not excluded

Erandy Ramirez\* and Dominik J. Schwarz†

*Fakultät für Physik, Universität Bielefeld, Universitätsstraße 25, Bielefeld 33615, Germany*

(Dated: September 15, 2009)

We present counterexamples to the claim that the  $\lambda\phi^4$  inflaton potential is excluded by recent cosmological data. Finding counterexamples requires that the actually observed primordial fluctuations are generated at the onset of the slow-roll regime of inflation. This set up for the initial conditions is therefore different from the usual scenario of chaotic inflation where inflation starts long before the observed fluctuations are created. The primordial power spectrum of “just enough” chaotic inflation violates scale-invariance in a way consistent with observations.

PACS numbers: 98.80.Cq

## I. INTRODUCTION AND BASIC IDEA

Cosmological inflation is considered to be the essential mechanism for setting the initial conditions of the standard cosmological scenario. At the same time it provides an explanation for the formation of structures in the Universe and there exists a collection of inflationary models whose predictions can be contrasted to observations. Constraining the space of allowed models of inflation is a major goal of CMB experiments such as WMAP and Planck.

One of the simplest and most popular models of inflation introduces just one self-interacting (real) scalar field  $\phi$  in the context of the chaotic scenario [1]. In this scenario the fluctuations of matter and space-time observed by means of the CMB temperature anisotropies are created long after the onset of inflation. Observable modes cross out of the Hubble horizon at 50 to 60 e-foldings before the end of inflation, whereas the total duration of chaotic inflation is  $\mathcal{O}(10^3)$  e-foldings. Thus the kinetic energy of the field plays no significant role for the observable fluctuations and the Universe is said to be well within the slow-roll regime. For the dynamical evolution of a particular model assumed to be in this stage, it is justified to apply the slow-roll approximation to the equations of motion and therefore to neglect the kinetic energy of the inflaton. It is within this picture that the  $\lambda\phi^4$  potential for inflation has been excluded by recent observations [2], although including a “curvaton” field relaxes the observational constraints on this potential [3].

Here we consider a different situation in which the total amount of inflation is not much more than 60 e-foldings. The onset of inflation is thus observable and therefore the effect of the kinetic energy is important. The resulting primordial power spectrum is not scale invariant, as the moment of the onset of the slow-roll regime distinguishes a scale. We find that a negative running of the spectral index is a feature of this scenario. This also means that the Universe undergoes only a small amount of inflation

before entering the slow-roll regime. Although not in accordance with the generic initial conditions of chaotic inflation, such a situation cannot be discarded on grounds of current analysis and observations.

This new scenario of “just enough” chaotic inflation seems to be generic if two fundamental energy scales are relevant in the very early Universe. The Planck scale  $M_p$  is the fundamental scale of quantum gravity. The notions of spatial curvature, expansion rate  $H$  and kinetic energy density  $\frac{1}{2}\dot{\phi}^2$  seem to be well defined and real quantities, at least up to that scale. However, this is less clear for the effective potential  $V$  of the inflaton. The effective potential carries all information about all the interactions of the inflaton except its gravitational ones. There exist examples of (low-energy) effective field theories whose potential becomes complex or show a singularity at some high-energy scale that is still well below the Planck scale.

The standard model of particle physics is one of these examples: The quartic self-coupling of the Higgs runs with energy. Except for a very heavy Higgs, the self-coupling decreases with increasing energy scale and can even become negative at high energy. The effective Higgs potential is real as long as the quartic coupling is positive, but becomes complex as soon as the self-coupling runs to negative values. Thus the standard model provides an example in which loop corrections give rise to an imaginary contribution of the effective potential. Such an imaginary contribution would give rise to the decay of the Higgs field. [4].

A fundamental energy scale  $M$  acting as an upper bound on the effective inflaton potential also shows up in supergravity models. The effective potential becomes too steep for inflation at some high energy scale. Although this mechanism gives rise to a low-energy potential that does not directly apply to the case we are considering here, it is an example of an upper bound for the validity of the inflaton potential [5].

Motivated by these examples, here we consider the possibility that there exists a second fundamental scale  $M < M_p$  that sets an upper bound for the validity of a description in terms of an effective potential  $V < M^4$ . Above this scale, the potential might be complex, giving rise to the decay of the inflaton. In the context of chaotic inflation the inflaton field takes values well

\*Electronic address: eramirez@physik.uni-bielefeld.de

†Electronic address: dschwarz@physik.uni-bielefeld.de

above the Planck scale, which is also the case in our scenario, nevertheless this will not prevent the potential to be of the order of  $M^4$  at the onset of inflation and before during its evolution. Thus just enough chaotic inflation follows generically from assuming that the Heisenberg uncertainty provides us with the initial conditions  $H \sim M_p$ ,  $\dot{\phi} \sim M_p^2$ , as in standard chaotic inflation, but  $V \sim M^4 \ll M_p^4$  as there exists a fundamental scale  $M < M_p$  above which the inflaton is unstable. As will be shown below, the current observational constraints are consistent with the assumption that this scale corresponds to that of grand unification theories  $M \sim M_{\text{GUT}}$ .

## II. $\phi^4$ INFLATION

We assume the homogeneity, isotropy and flatness of the Universe from the onset of inflation, although this is certainly a very rough approximation only. The equations of motion are then

$$H^2 = \frac{1}{3M_p^2} \left( \frac{\dot{\phi}^2}{2} + V \right), \quad (1)$$

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0;$$

a prime denotes a derivative with respect to the field,  $M_p \equiv m_p/\sqrt{8\pi} \approx 2.4 \times 10^{18}$  GeV is the reduced Planck mass. For the numerical analysis we rewrite them as

$$\frac{dH}{d\phi} = \frac{1}{M_p^2} \sqrt{\frac{3M_p^2 H^2 - V}{2}}, \quad (2)$$

$$\frac{dN}{d\phi} = -\frac{H}{\sqrt{2(3M_p^2 H^2 - V)}}.$$

Our convention is  $\dot{\phi} < 0 \Rightarrow H' > 0$  and  $H \equiv dN/dt$  with  $t$  cosmic time, therefore  $dN > 0$  as  $dt > 0$ .

For  $V = \lambda\phi^4/24$ , a value of  $\lambda \sim 10^{-12}$  is required to get at least the normalization of the inflationary fluctuations right.

Below we make use of the horizon-flow functions  $\epsilon_n$  [6], which are a generalization of the slow-roll parameters and are defined recursively:

$$\epsilon_0 \equiv \frac{H_i}{H}, \quad \epsilon_{m+1} \equiv \frac{1}{\epsilon_m} \frac{d\epsilon_m}{dN}, \quad m \geq 0. \quad (3)$$

$H_i$  refers to the initial Hubble rate (at  $N = 0$ ). For single scalar field models of inflation, the first horizon flow function has the simple interpretation to be proportional to the ratio of the kinetic energy density to the total energy density of the field,

$$\epsilon_1 = 3 \frac{\frac{1}{2}\dot{\phi}^2}{\frac{1}{2}\dot{\phi}^2 + V}. \quad (4)$$

In the slow-roll approximation, the horizon-flow functions can be related to the usual slow-roll parameters [7].

### A. Initial Conditions

In order to solve the horizon and flatness problems at or close to the Planck scale, we would need at least 50 e-foldings. For the  $\lambda\phi^4$  potential we can use the slow-roll approximation to estimate the amount of e-foldings to the end of inflation as  $\Delta N \simeq (\phi^2 - \phi_e^2)/(8M_p^2)$ , with  $\phi_e \simeq \sqrt{8}M_p$ . Thus we need to specify the initial conditions of inflation at  $\phi_i \geq 20M_p$ .

In the spirit of chaotic inflation we would expect that  $H_i \sim M_p$ ,  $\dot{\phi} \sim M_p^2$  and  $V \sim M_p^4$ . As mentioned before, we explore here a situation where perturbations are evaluated just before the onset of the slow-roll regime and thus must limit the potential to  $V_i \sim M^4 \ll M_p^4$ . This implies that  $\epsilon_{1i} \approx 3$ , since  $V \ll \dot{\phi}^2$ . Inflation requires  $\epsilon_1 < 1$ . This means that the Universe starts in a kinetic energy dominated regime. Due to the Hubble drag the kinetic energy density decays quickly and the dynamics becomes dominated by the potential energy density. Inflation starts and a slow-roll behaviour is approached. One has the freedom to choose  $0 < \epsilon_{1i} < 3$ ; in particular,  $\epsilon_{1i} \simeq 2.9$  is compatible with chaotic inflation since the evolution of the system will drive it rapidly to the slow-roll attractor [8].

For the initial Hubble rate and the number of e-foldings, we use:

$$H_i = \frac{1}{\sqrt{3 - \epsilon_{1i}}} \sqrt{\frac{V_i}{M_p^2}}, \quad N_i = 0. \quad (5)$$

Starting from an initial condition as specified in the preceeding paragraph and the use of the horizon flow functions, allow us to find different results from the ones already reported for the  $\lambda\phi^4$  potential. The exclusion of  $\lambda\phi^4$  inflation is obtained in terms of parameters that measure the ratio of the potential to its derivatives,  $\epsilon_V \equiv M_p^2/2 (V'/V)^2$ ,  $\eta_V \equiv M_p^2 V''/V$  [7]. In doing so, one implicitly assumes the validity of the slow-roll approximation and therefore omits the contribution of the kinetic energy before inflation starts, which in time excludes different possibilities for its set up.

In Figure 1 we can appreciate this difference; the region of slow-roll in both trajectories is the same, except that the horizon flow functions account for the full dynamics. The evolution of  $\epsilon_1$  and  $\epsilon_2$  agrees with that of  $\epsilon_V$  and  $4\epsilon_V - 2\eta_V$  within the slow-roll regime, but differs significantly at the onset of inflation and by a smaller amount towards the end.

### B. Spectrum of Density Perturbations

An important condition that any inflationary model must fulfill is to produce the observed amplitude of the power spectrum of scalar perturbations

$$\Delta_R^2(k) \equiv \frac{k^3 P_R(k)}{2\pi^2}. \quad (6)$$

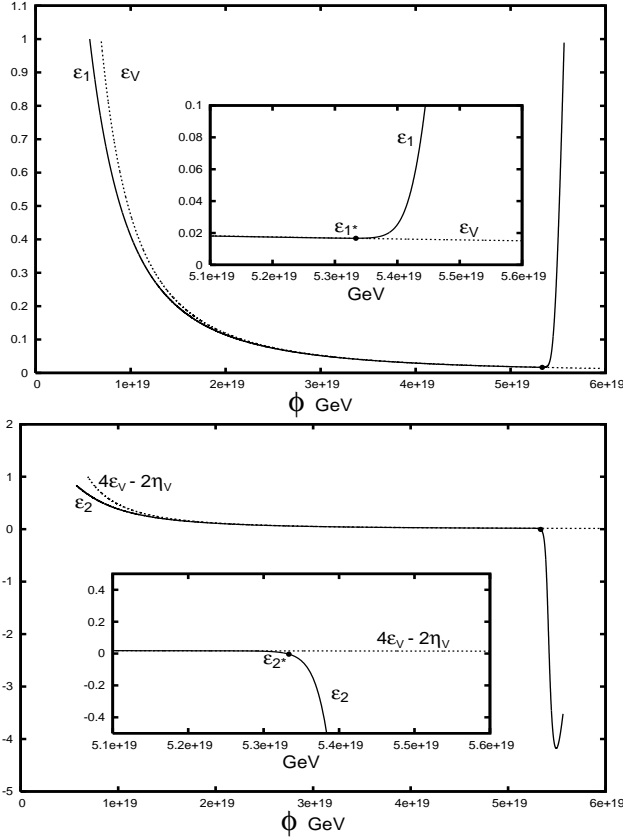


FIG. 1: Horizon flow functions and potential slow-roll parameters for an initial condition of  $\epsilon_1 = 2.9$  and initial field value  $\phi_i = 24.4M_p$ . The  $\bullet$  indicates the moment of time when modes observed in the CMB cross the horizon and where perturbations are evaluated.

The best-fit amplitude at the pivot scale of  $k_* = 0.002/\text{Mpc}$  was reported as  $\Delta_R^2 = (2.445 \pm 0.096) \times 10^{-9}$  by the WMAP team [2]. We follow the convention of WMAP5 and approximate the amplitude by a power-law:

$$\Delta_R^2(k) \approx \left( \frac{H_*^2}{8\pi^2 M_p^2 \epsilon_{1*}} \right) A_* \left( \frac{k}{k_*} \right)^{n_s - 1}, \quad (7)$$

where  $n_s - 1$  is the spectral tilt. A  $*$  indicates the scale at which perturbations are evaluated.  $A$  denotes the first-order correction to the amplitude of the power spectrum in terms of the horizon-flow parameters [9]:

$$A \approx 1 - 2(C + 1)\epsilon_1 - C\epsilon_2, \quad (8)$$

where  $C \equiv \gamma_E + \ln 2 - 2 \simeq -0.7296$  and we have kept contributions of  $\mathcal{O}(\epsilon_i)$  only.

The approximation given by Eq. (7) does not consider the inclusion of running of the spectral index, in our results we obtain running and therefore make use of the power-law approximation for this situation given by:

$$\Delta_R^2(k) \approx \left( \frac{H_*^2}{8\pi^2 M_p^2 \epsilon_{1*}} \right) A_* \left( \frac{k}{k_*} \right)^{n_{s*} - 1 + \frac{1}{2} \frac{dn_{s*}}{d \ln(k/k_*)} \ln(k/k_*)}, \quad (9)$$

where  $\frac{dn_{s*}}{d \ln(k/k_*)}$  is the running of the spectral index.

The tensor-to-scalar ratio  $r$  and the spectral tilt are at leading order in the horizon-flow functions given by:

$$\begin{aligned} r &\approx 16\epsilon_1, \\ n_s - 1 &\approx -2\epsilon_1 - \epsilon_2. \end{aligned} \quad (10)$$

### C. End of Inflation and Pivot Scale

The end of inflation is given by the condition  $\epsilon_1 = 1$  that fixes the total number of e-foldings  $N_e$ , and the value of the field at the end of inflation  $\phi_e$ . Let  $\Delta N_* \equiv N_e - N_*$  be the number of e-foldings between horizon crossing of the pivot scale  $k_*$  and the end of inflation.

For the comparison of the theoretical power spectra with observations, we choose values of  $\Delta N_*$  within a certain interval. Then we calculate the pivot scale  $k_* = a_* H_*$ . In order to do this, one needs a model of reheating, and for the moment we assume sudden reheating to a radiation-dominated Universe at the end of inflation, which means that our values are upper estimates for  $\Delta N_*$ .

We know that,

$$\epsilon_1 = 1 \Rightarrow \dot{\phi}_e^2 = V_e, \quad H_e^2 = \frac{1}{2M_p^2} V_e = \frac{1}{3M_p^2} \rho_r \quad (11)$$

with  $\rho_r = (\pi^2/30)gT_r^4$  and  $T_r^4 = \frac{45}{\pi^2} \frac{V_e}{g}$  being the energy density and temperature at reheating.  $g$  is the effective number of relativistic helicity degrees of freedom. Above  $T \sim 100$  GeV we take the value  $g = 106.75$ . Therefore, for the pivot scale  $k_* = a_* H_*$  we have from  $k(\phi)/k_* = e^{\Delta N} H(\phi)/H_*$ , where  $\Delta N = N(\phi) - N_*$  and with  $a_0 = 1$ :

$$\begin{aligned} k_* &= e^{-\Delta N_*} H_* \frac{a_e}{a_0} a_0 = e^{-\Delta N_*} H_* (1+z)^{-1} \\ &= 500 e^{-\Delta N_*} H_* \frac{T_{\nu 0}}{T_{reh}} (0.002 \text{Mpc}^{-1}), \end{aligned} \quad (12)$$

where the redshift to the end of inflation is given by the ratio of today's neutrino temperature to the reheating temperature.

### D. Methodology

Our scenario of inflation is controlled by three parameters: the initial value of the inflaton field,  $\phi_i = a M_p$ , the initial value of  $\epsilon_1$  and the scale at which one evaluates the perturbations, namely, the value of  $\Delta N_*$ . This last point has to be considered carefully since an initial value of  $\epsilon_{1i} > 1$  gives less inflation on the whole trajectory than a value of  $\epsilon_{1i} \approx 0$ . Additionally, we want to evaluate perturbations at the point when  $|\epsilon_2| \ll 1$  and  $\epsilon_2 < 0$ . The reason being that the case  $0 < \epsilon_2 < 1$  gives the values already excluded by the WMAP plus baryonic acoustic oscillations plus super novae (WMAP+BAO+SN) analysis. A  $\Delta N_*$  lying on the region where both trajectories in Fig. 1 are the same, will resort to values of  $r$  and  $n_s$  already excluded.

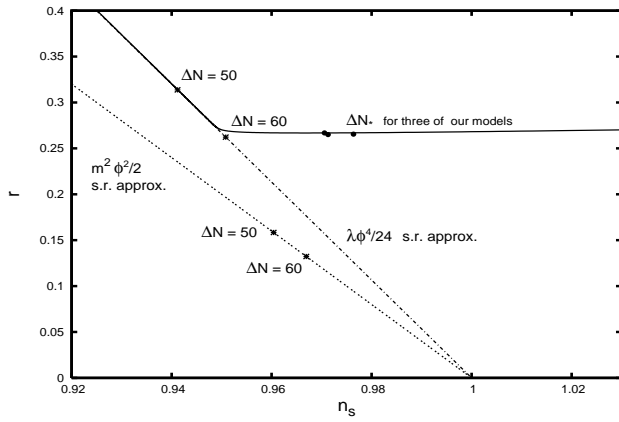


FIG. 2: Tensor-to-scalar ratio  $r$  versus the spectral index  $n_s$  for the three models detailed in Table I and the trajectory of the second of them. Our examples violate the slow-roll approximation. For comparison the slow-roll trajectories of the  $\lambda\phi^4$  and the  $m^2\phi^2$  potentials are shown.

### III. RESULTS

For our results we set  $\lambda$  to  $10^{-12}$ . The values for  $\mathbf{a} = \phi_i/M_p$ ,  $\epsilon_1$  and  $\Delta N_*$  are chosen within the following intervals:

$$\epsilon_1 \in [0.5, 2.9], \quad \Delta N_* \in [58, 65], \quad \mathbf{a} \in [20, 35]. \quad (13)$$

The interval of values for  $\mathbf{a}$  assures that the value of  $\phi_i$  is well above  $3M_p$  (the end of inflation) and allows for at least 50 e-foldings, corresponding to the lower limit of the interval. The upper limit takes into account that during the onset of inflation the short epoch of fast roll does not give rise to an exponential expansion of the Universe.

The location in parameter space for three cases are shown in Figure 2 and their predictions given explicitly in Table I. From the figure, it is possible to appreciate that the values for the tensor-to-scalar ratio  $r$  are approximately the same as in the usual slow-roll approximation, since the interval of values of  $\Delta N_*$  is concentrated around 60 e-foldings. The crucial difference is given by the value of  $\epsilon_{2*}$ , which at this level of approximation enters only in the spectral tilt and 'shifts' the points towards the right. These results cannot be obtained in a model that considers initial conditions for the system in the slow-roll regime. Our results for the spectral index are consistent with the bounds of WMAP5 for Running+Tensors at the  $2\sigma$  level, the running and the tensor-to-scalar ratio as well as the amplitude of the spectrum are inside the  $1\sigma$  interval.

For the second of the examples shown in Table I the spectrum produced and the approximation given by Eq. (7) and Eq. (9) are presented in Fig. 3. The zoom in the figure gives an idea about how the spectrum looks in the actually observed region of the power spectrum.

The difference between the spectra produced using  $\epsilon_1$ ,  $\epsilon_2$  and  $\epsilon_V$ ,  $\eta_V$  can be appreciated in Fig. 4, the region shown corresponds to the onset of the slow-roll

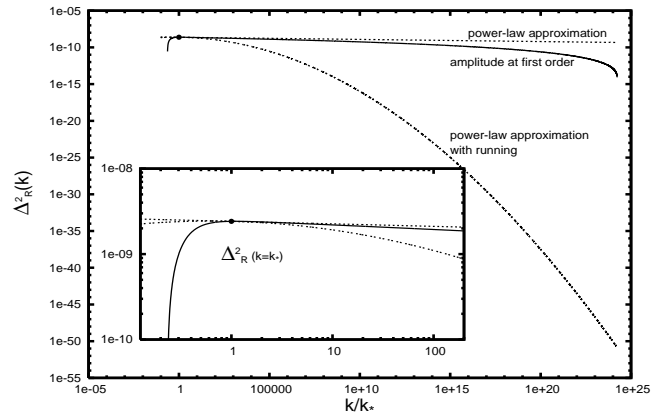


FIG. 3: Power spectrum of scalar fluctuations at first order in the horizon flow functions, its power-law approximation without running, and the power-law approximation including running. The  $\bullet$  indicates where perturbations are evaluated.

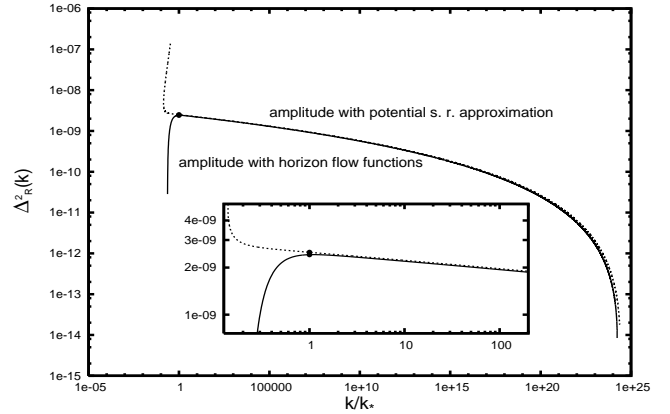


FIG. 4: Power-spectrum of scalar fluctuations: the upper curve corresponds to using  $\epsilon_V$ ,  $\eta_V$  and the leading slow-roll approximation. The lower-curve is the result of using  $\epsilon_1$ ,  $\epsilon_2$ . The dots indicate where perturbations are evaluated.

regime during which the behavior of both is indistinguishable. Before the point where fluctuations are evaluated at  $k < k_*$ , there is however, considerable difference between both of them as a consequence of the dynamics.

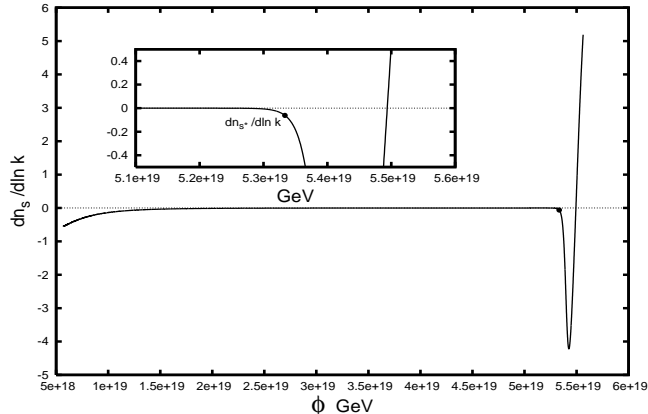
In Fig. 5, the running of the spectral index,

$$\frac{dn_s}{d\ln(k/k_*)} \approx -\epsilon_2 (2\epsilon_1 + \epsilon_3) \quad (14)$$

shows that perturbations are evaluated when the system has not yet arrived to the region when the spectrum can be approximated completely by a power-law. The value of the running is negative for all cases and of order  $\mathcal{O}(10^{-2})$ . This can be explained by noting that at the point where perturbations are evaluated,  $\epsilon_2$  has not yet arrived to the slow-roll regime [6, 10]. In fact it is going to change sign and thus higher-order horizon flow functions will diverge at that point, which just means that the slow-roll approximation cannot be applied. Strictly speaking we should solve the mode equations for the per-

TABLE I: Examples of “just enough” chaotic inflation for  $V = \lambda\phi^4/24$  and  $\lambda = 10^{-12}$ .

$\epsilon_{1,i}$	$\phi_i/M_p$	$V_i/M_p^4$	$N_T$	$\Delta N_*$	$\epsilon_{1*}$	$\epsilon_{2*}$	$\epsilon_{3*}$	$r_*$	$n_{s*}$	$dn_{s*}/d\ln k$	$H_*^2 A/(8\pi^2 M_p^2 \epsilon_{1*})$	$k_*$
0.77	22.832	$1.1 \times 10^{-8}$	62.96	60.50	$1.659 \times 10^{-2}$	$-9.555 \times 10^{-3}$	-8.176	0.27	0.98	$-7.78 \times 10^{-2}$	$2.45 \times 10^{-9}$	0.010
2.9	24.4	$1.1 \times 10^{-8}$	62.69	60.08	$1.668 \times 10^{-2}$	$-3.939 \times 10^{-3}$	-15.629	0.27	0.97	$-6.143 \times 10^{-2}$	$2.42 \times 10^{-9}$	0.015
2.2	23.5	$1.2 \times 10^{-8}$	63.10	60.50	$1.656 \times 10^{-2}$	$-4.42 \times 10^{-3}$	-14.178	0.27	0.97	$-6.25 \times 10^{-2}$	$2.47 \times 10^{-9}$	0.010

FIG. 5: Running of the spectral index as a function of the inflaton field for  $\epsilon_{1i} = 2.9$  and  $\phi_i = 24.4$ . The  $\bullet$  indicates the moment of time when the pivot wave number  $k_*$  crosses the horizon and where perturbations are evaluated.

turbations numerically to obtain a more reliable result. However, it seems to us that for a first study of our scenario the current level of sophistication is good enough.

The values of the scale  $k_*$  at which perturbations are evaluated for our results are bigger by a factor of 5 than the pivot scale used by WMAP5. One has to bear in mind the uncertainty due to assuming sudden reheating for obtaining these numbers.

The initial value of the potential at the Planck scale is determined by the quartic coupling  $\lambda$  and  $\phi_i$ , these two numbers set the potential to 8 orders of magnitude below  $M_p^4$ . This confirms the scenario described above, which differs from the usual “chaotic” initial conditions where kinetic and potential energies are of the same order of magnitude initially. We have seen that the initial condition of  $\epsilon_1$  very close to 3 is needed in order to also have  $H_i$  of the order of the Planck mass. This initial condition has also allowed us to obtain acceptable values for the inflationary observables.

However, from Table I it is possible to see that an initial  $H_i \sim M_p$  is not a necessary requirement to ensure acceptable  $r$  and  $n_s$ ; an initial value as low as 0.77 for  $\epsilon_1$  gives also acceptable models. Therefore, an initial  $\epsilon_1$  that gives convenient values for the observables need not be arbitrarily close to 3, but only outside of the slow-roll regime. Nevertheless, in our scenario a value of  $\epsilon_{1i} \approx 3$ , seems most natural to us.

#### IV. CONCLUSIONS

From the results shown in Figure 2 and Table I, we conclude that the  $\lambda\phi^4$  inflaton potential is not excluded. This conclusion is obtained within a set up that allows for the existence of a fundamental maximal scale of the inflaton potential  $V \sim M^4$ , which is well below the Planck scale  $M_p^4$  at the beginning of inflation. Such a scale arises naturally in the context of the standard model of particle physics and some of its suggested extensions. Loop corrections to the effective potential might become imaginary, leading to a destabilization of the inflaton field. In supergravity theories of inflation the potential might become too steep. Both instances would define a scale  $M < M_p$ . Below  $M$  all interactions apart from gravity are taken into account by the effective potential. From the results of Table I, it is consistent to assume  $M$  to be the scale of grand unification (GUT scale), which is at least a pleasing coincidence.

The use of the horizon flow functions instead of the potential slow-roll parameters allowed us to follow the cosmological evolution from the onset of inflation for the model considered here. The difference between both approaches has also been shown in more complicated models of inflation like the hybrid inflation scenario [11].

As is obvious from Fig. 4, the power-law approximation either with running or without it, is not describing the actual power spectrum very well. Both of the approximations overestimate the power at large length scales, the approximation without running is closer to the true amplitude on small scales. The approximation with running significantly underestimates the power at small scales. This implies that for a detailed analysis of the epoch of the onset of slow-roll, we cannot just take the published fits from the WMAP analysis on  $\Delta_R^2$ ,  $n_s$  and  $r$ , but we should run a new Markov chain Monte Carlo integration. The fact that we find values of correct order of magnitude makes us confident that a more refined analysis will also allow us to find a good fit. The calculation of the power spectrum itself should be based on a numerical mode-by-mode integration, as the expansion in horizon flow functions that we are using here is at best an estimate of the true power spectrum. The usual slow-roll approximation, as argued above, is even worse.

As already noted, we find a power spectrum that is not featureless. The amount of running is in agreement with observations. On top of that, we find a suppression of power on the largest scales. This suppression could be linked to the lack of CMB correlations on large angular

scales [12, 13]. In order to explain the lack of power on large scales it was proposed before to consider a period of fast-roll at the beginning of inflation and to have only 60 to 65 e-foldings of expansion [14, 15] (very much like in our scenario), or to assume a fast-roll epoch in between two epochs of slow-roll [16]. The difference to our work is however, that different inflationary potentials have been used and it seems to us that our set-up is more natural. Apart from the lack of large scale correlation, the observed alignment of quadrupole and octupole [17, 18] could be a remainder of the not yet perfect statistical isotropy at the onset of inflation. A detailed study of those aspects is beyond the scope of this work.

This alternative set up for inflation, based on the existence of a second fundamental scale  $M < M_p$ , leads us to the consideration of models that are not properly described by slow-roll inflation (but nevertheless contain an

epoch of slow-roll). To exclude a specific model of inflation by means of observations, one needs to test whether the fluctuations are evaluated in the slow-roll regime. Improved analysis of available and upcoming data from WMAP and from the Planck satellite will allow us to probe the new scenario of "just enough" chaotic inflation.

## V. ACKNOWLEDGMENTS

We thank S. Clesse, M. Einhorn, G. Isidori, T. Jones, G. Moore, H. Solis, L. Sriramkumar A. Strumia and T. Takahashi for interesting discussions and/or hints to the literature. E. R. was supported by DAAD research grant A/07/26003 and Conacyt grant 79045.

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- [1] A. D. Linde, Phys. Lett. B **129** (1983) 177.
  - [2] E. Komatsu *et al.* (WMAP Collaboration), Astrophys. J. Suppl. **180**, 330 (2009).
  - [3] T. Moroi, T. Takahashi and Y. Toyoda, Phys. Rev. D **72**, 023502 (2005). K. Ichikawa, T. Suyama, T. Takahashi and M. Yamaguchi, Phys. Rev. D **78**, 023513 (2008).
  - [4] C. Ford, D. R. T. Jones, P. W. Stephenson and M. B. Einhorn, Nucl. Phys. B **395**, 17 (1993).
  - [5] M. Kawasaki, M. Yamaguchi and T. Yanagida, Phys. Rev. Lett. **85**, 3572 (2000); K. Suzuki and T. Watari, Phys. Lett. B **506**, 231 (2001); R. Kallosh and A. Linde, J. Cosmol. Astropart. Phys. 04 (2007) 017.
  - [6] D. J. Schwarz, C. A. Terrero-Escalante and A. A. Garcia, Phys. Lett. B **517**, 243 (2001).
  - [7] A. R. Liddle, P. Parsons and J. D. Barrow, Phys. Rev. D **50**, 7222 (1994).
  - [8] A. D. Linde, "Particle Physics and Inflationary Cosmology", Harwood Academic Publishers, Chur, Switzerland, (1990), chapter 9.
  - [9] S. M. Leach, A. R. Liddle, J. Martin and D. J. Schwarz, Phys. Rev. D **66**, 023515 (2002).
  - [10] D. J. Schwarz and C. A. Terrero-Escalante, J. Cosmol. Astropart. Phys. 08 (2004) 003.
  - [11] S. Clesse and J. Rocher, Phys. Rev. D **79**, 103507 (2009).
  - [12] G. Hinshaw *et al.*, Astrophys. J. **464**, L25 (1996); D. N. Spergel *et al.* (WMAP Collaboration), Astrophys. J. Suppl. **148**, 175 (2003).
  - [13] C. J. Copi, D. Huterer, D. J. Schwarz and G. D. Starkman, arXiv:0808.3767 [astro-ph].
  - [14] C. R. Contaldi, M. Peloso, L. Kofman and A. Linde, J. Cosmol. Astropart. Phys. 07 (2003) 002.
  - [15] G. Nicholson and C. R. Contaldi, J. Cosmol. Astropart. Phys. 01 (2008) 002.
  - [16] R. K. Jain, P. Chingangbam, J. O. Gong, L. Sriramkumar and T. Souradeep, J. Cosmol. Astropart. Phys. 01 (2009) 009.
  - [17] A. de Oliveira-Costa, M. Tegmark, M. Zaldarriaga and A. Hamilton, Phys. Rev. D **69**, 063516 (2004).
  - [18] D. J. Schwarz, G. D. Starkman, D. Huterer and C. J. Copi, Phys. Rev. Lett. **93**, 221301 (2004); C. J. Copi, D. Huterer, D. J. Schwarz and G. D. Starkman, Phys. Rev. D **75**, 023507 (2007).